# MTH 3210 Written Homework 3

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## Section 2.1: 3, 4, 8

**Question 3.**

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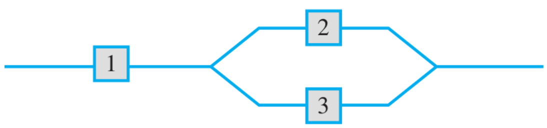
Three components are connected to form a system as shown in the accompanying diagram. Because the components in the 2-3 subsystem are connected in parallel, that subsystem will function if at least one of the two individual components function. For the entire system to function, component 1 must function and so must the 2-3 subsystem.




The experiment consists of determining the condition of each component [$S$ (success) for functioning component and $F$ (failure) for a non-functioning component.]

**a. Which outcomes are contained in the event $A$ that exactly two of the three components function?**
The following outcomes have exactly two functioning components.

$$A = \{SSF, FSS, SFS\}$$

**b. Which outcomes are contained in the event $B$ that at least two of the components function?**
The following outcomes have at least two functioning components.

$$B = \{SSF, FSS, SSS, SFS\}$$

**c. Which outcomes are contained in the event $C$ that the system functions?**
The following outcomes allow the system to function.

$$C = \{SFS, SSF, SFS\}$$

**d. List outcomes in $C', A \cup C, A \cap C, B \cup C,$ and $B \cap C$.**

$$\begin{aligned} C' &= \{SFF, FFF, FFS, FSF\} \\ A \cup C &= \{SSF, FSS, SFS, SSS\} \\ A \cap C &= \{SSF, SFS\} \\ B \cup C &= \{SSF, FSS, SSS, SFS\} \\ A \cap C &= \{SSS, SSF, SFS\} \end{aligned}$$

**Question 4.**

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Each of a sample of four home mortgages is classified as fixed rate ($F$) or variable rate ($V$).

**a. What are the 16 outcomes in $S$?**
The 16 outcomes that make up our sample space are as follows:

$$S = \left\{ \begin{array}{llll} FFFF, & FFFV, & FFVV, & FVVV, \\ FVVF, & FVFF, & FVFV, & FFVF, \\ VVVV, & VVVF, & VVFF, & VFFF, \\ VFFV, & VFVV, & VFVF, & VVFV \end{array} \right\}$$

**b. What outcomes are in the event that exactly three of the selected mortgages are fixed rate?**
The event ($A$) that satisfies these conditions has the following outcomes:

$$A = \{FFFV,\ FVFF,\ FFVF,\ VFFF\}$$

**c. Which outcomes are in the event that all four mortgages are of the same type?**
The event ($B$) that satisfies these conditions has the following outcomes:

$$B = \{FFFF,\ VVVV\}$$

**d. Which outcomes are in the event that at most one of the four is a variable-rate mortgage?**
The event ($C$) that satisfies these conditions has the following outcomes:

$$C = \{FFFF,\ FFFV,\ FVFF,\ FFVF,\ VFFF\}$$

**e. What is the union of events $B$ and $C$? What is the intersection of these two events?**
The set of outcomes that are in at least one of $B$ or $C$ is as follows:

$$B \cup C = \{FFFF,\ FFFV,\ FVFF,\ FFVF,\ VFFF,\ VVVV\}$$

The set of outcomes that are in both $B$ and $C$ is as follows:

$$B \cap C = \{FFFF\}$$

**f. What is the union of events $A$ and $B$? What is the intersection of these two events?**
The set of outcomes that are in at least one of $A$ or $B$ is as follows:

$$A \cup B = \{FFFV,\ FVFF,\ FFVF,\ VFFF,\ FFFF,\ VVVV\}$$

The set of outcomes that are in both $A$ and $B$ is an empty set.

$$A \cap B = \varnothing$$

**Question 8.**

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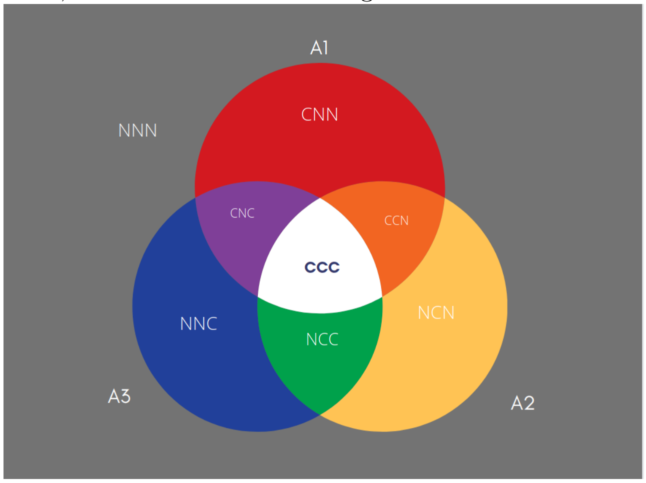
An engineering construction firm is currently working on power plants at three different sites. Let $A_i$ denote the event that the plant at site $i$ is completed by the contract date. Use the operations of union, intersection, and complementation to describe each of the following events in terms of $A_1, A_2$, and $A_3$, draw a venn diagram, and shade the region corresponding to each one.

First, let's write out our entire sample space. The notation I'll be using will be $C$ for completed and $N$ for not completed.

$$S = \left\{ \begin{array}{c} CCC,\ CCN,\ CNN,\ CNC, \\ NNN,\ NNC,\ NCC,\ NCN \end{array} \right\}$$

Next, let's create our Venn diagram.




**a. At least one plant is completed by the contract date.**
This event can be described as the union between $A_1$, $A_2$, and $A_3$. In set notation this would be $(A_1 \cup A_2 \cup A_3)$. In terms of the Venn diagram, that would be everything excluding the grey outside area.

**b. All plants are completed by the contract date.**
This event includes one single outcome, that being encompased by the white section in the center of the diagram. In terms of set notation that would be $(A_1 \cap A_2 \cap A_3)$.

**c. Only the plant at site 1 is completed by the contract date.**
The red section of the Venn diagram would be the only section to fit this circumstance. I believe this can be described as $A_1$ intersecting with the intersection of $A_2'$ and $A_3'$. In other words: $(A_1 \cap (A_2' \cap A_3'))$

**d. Exactly one plant is completed by the contract date.**
Using the Venn diagram, that would include the red, yellow and blue sections. The answer that came to mind is less than graceful but should work. I will neglect writing it out in words and stick entirely to set notation.
$(A_1 \cap A_2' \cap A_3') \cup (A_2 \cap A_1' \cap A_3') \cup (A_3 \cap A_1' \cap A_2')$

**e. Either the plant at site 1 or both of the other two plants are completed by the contract date.**
To explain this, our scenario here would include all 4 outcomes in $A_1$ alongside the intersection of $A_2$ and $A_3$. With the Venn diagram, that would be the red, orange, purple, white, and green sections. In set notation, that would be:
$A_1 \cup (A_2 \cap A_3)$

## Section 2.2: 12-15, 19, 21-22, 26

**Question 12.**

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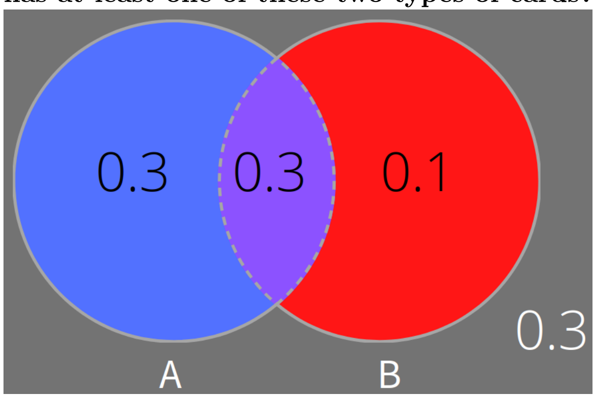
Consider randomly selecting a student at a large university, and let $A$ be the event that the selected student has a Visa card and $B$ be the analogous event for MasterCard. Suppose that $P(A) = .6$ and $P(B) = .4$.

**a. Could it be the case that $P(A \cap B) = .5$? Why or why not?** It cannot be the case. The intersection of the two events cannot possibly be greater than the value of one of the individual events. The highest $P(A \cap B)$ could be is 0.4, and that is only if $B$ completely intersected with $A$.

**b. From now on, suppose that $P(A \cap B) = .3$. What is the probability that the selected student has at least one of these two types of cards?**




The probability of this can be found by simply adding up the three probabilities within the circles. That would give us $P(A \cup B) = .7$

**c. What is the probability that the selected student has neither type of card.**
This one can be found by simply subtracting the .7 we just got from 1. If a student doesn't have at least one of the two cards, then they have neither. Simple as that. This can be represented as $P(A' \cap B') = .3$.

**d. Describe, in terms of $A$ and $B$, the event that the selected student has a Visa card but not a MasterCard, and then calculate the probability of the event.**
This can be described as the blue circle minus the intersecting part in the middle. We can describe that as $P(A \cap B') = .3$. We know the value is .3 because the intersection between the two events is .3, and $.6 - .3 = .3$.

**e. Calculate the probability that the selected student has exactly one of the two types of cards.**
This one is easy. We simply add the two outer circles together and ignore the intersection. As such, that gives us $P((A \cup B) - A \cap B) = .4$

**Question 13.**

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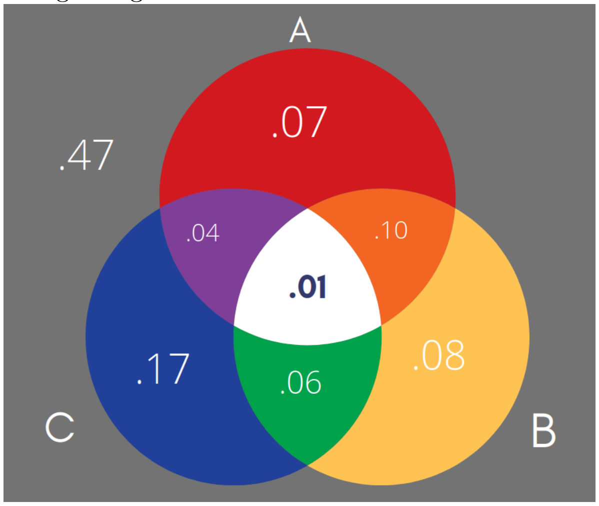
A computer consulting firm presently has bids out on three projects. Though the book refers to awarded projects using $A_i$, let us instead use $A, B$ and $C$ for the respective three rewards to improve readability. Suppose that $P(A) = .22$, $P(B) = .25$, $P(C) = .28$, $P(A \cap B) = .11$, $P(A \cap C) = .05$, $P(B \cap C) = .07$, $P(A \cap B \cap C) = .01$. Express in words the following events, and compute the probability of each event.

$$P(A) = .22$$
$$P(B) = .25$$
$$P(C) = .28$$
$$P(A \cap B) = .11$$
$$P(A \cap C) = .05$$
$$P(B \cap C) = .07$$
$$P(A \cap B \cap C) = .01$$

Using this given information I was able to construct this Venn diagram.




**a.** $A \cup B$
This event would be when either project A or project B gets an award.
$P(A \cup B) = 0.07 + 0.10 + 0.01 + 0.08 + 0.06 = 0.32$

**b.** $A' \cap B'$
This event is the intersection of not A and not B. To break this down. Not A involves the yellow, green, blue and grey sections. Not B involves the blue, purple, red and grey sections. The intersection here is what is in both of these sets. That is, blue and grey. So, this event is when either only Project C gets an award, or none of the projects get an award.
$P(A' \cap B') = .47 + .17 = 0.64$

**c.** $A \cup B \cup C$
This event is any of the three projects getting an award. It encompasses the whole interior of the Venn diagram. As such: $P(A \cup B \cup C) = 1 - .47 = .53$

**d.** $A' \cap B' \cap C'$
This event is none of the three projects getting an award. It is the outside portion of the Venn diagram. As such, $P(A' \cap B' \cap C') = .47$

**e.** $A' \cap B' \cap C$

This event is neither A or B getting an award, but C getting an award. It is only the blue section of the Venn diagram. $P(A' \cap B' \cap C) = .17$

**f.** $(A' \cap B') \cup C$

This event is two parts. The first is the intersection of not A and not B, which is still blue and grey as discovered in part b. We then get the union of that with C, which gives us blue, green, white and purple. So we get the following probability.
$P((A' \cap B') \cup C) = .47 + .17 + .04 + .01 + .06 = .75$

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**Question 14.**

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Suppose that 55% of all adults regularly consume coffee, 45% regularly consume carbonated soda, and 70% consume at least one of these two products.

**a. What is the probability that a randomly selected adult regularly consumes both coffee and soda?**

Let C represent coffee drinkers and S represent soda drinkers. We can use the general addition rule here and rearrange it to solve for the intersection we need. We know that $P(C) = .55$, $P(S) = .45$ and that $P(C \cup S) = .70$.

$$P(C \cup S) = P(C) + P(S) - P(C \cap S)$$
$$P(C \cap S) = P(C) + P(S) - P(C \cup S)$$
$$P(C \cap S) = .45 + .55 - .70$$
$$P(C \cap S) = .3$$

**b. What is the probability that a randomly selected adult doesn't regularly consume at least one of these two products?**

This sets of outcomes involves the outside area of a theoretical Venn diagram we might make here. We can solve for that by using a variation of the complement rule.

$$P(C' \cap S') = 1 - P(C \cup S)$$
$$P(C' \cap S') = .3$$

**Question 15.**

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Consider the type of clothes dryer (gas or electric) purchased by each of five different customers at a certain store.

**a. If the probability that at most one of these purchases an electric dryer is 0.428, what is the probability that at least two purchase an electric dryer?**
This question is quite straightforward. We only have two options, one being at most one, the other being at least two. That means these two outcomes add up to 1. So, subtract .428 from 1. The probability that at least two purchase an electric dryer is .572.

**b. If $P$(all five purchase gas)$= 0.116$ and $P$(all five purchase electric)$= 0.005$, what is the probability that at least one of each type is purchased?**
Since all the other options we have are separate from the two given, we just subtract the two given probabilities from 1 and get our answer. The probability that at least one of each type is purchased is 0.879.

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**Question 19.**

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Human visual inspection of solder joints on printed circuit boards can be very subjective. Part of the problem stems from the numerous types of solder defects (e.g., pad non-wetting, knee visibility, voids) and even the degree to which a joint possesses one or more of these defects. Consequently, even highly trained inspectors can disagree on the disposition of a particular joint. In one batch of 10,000 joints, inspector A found 724 that were judged defective, inspector B found 751 such joints, and 1159 of the joints were judged defective by at least one of the inspectors. Suppose that one of the 10,000 joints is randomly selected.

**a. What is the probability that the selected joint was judged to be defective by neither of the two inspectors?** Our given information here is as follows. $P(A) = .0724$, $P(B) = .0751$ and $P(A \cup B) = .1159$. Since all joints that were not found defective by the two inspectors encompasses everything outside of our given union, we can simply subtract from 1. $1 - .1159 = .8841$.

**b. What is the probability that the selected joint was judged to be defective by inspector B but not by inspector A?** For this we can just use the probability of our given union but with the probability of A removed.

$$P(B \cap A') = P(A \cup B) - P(A)$$
$$P(B \cap A') = .1159 - .0724$$
$$P(B \cap A') = .0435$$

**Question 21.**

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An insurance company offers four different deductible levels—none, low, medium, and high—for its homeowner's policyholders and three different levels—low, medium, and high—for its automobile policyholders. The accompanying table gives proportions for the various categories of policyholders who have both types of insurance. For example, the proportion of individuals with both low homeowner's deductible and low auto deductible is .06 (6% of all such individuals).

| | **Homeowner's** | | | |
|---|---|---|---|---|
| **Auto** | **N** | **L** | **M** | **H** |
| **L** | .04 | .06 | .05 | .03 |
| **M** | .07 | .10 | .20 | .10 |
| **H** | .02 | .03 | .15 | .15 |

Suppose an individual having both of policies is randomly selected.

**a. What is the probability that the individual has a medium auto deductible and a high homeowner's deductible?**
For this we can simply look at the table to find our value. The section that meets our criteria gives us $\boxed{.10}$ for our probability.

**b. What is the probability that the individual has a low auto deductible? A low homeowner's deductible?**
For these two answers we can simply add up the proportions in the low auto section and then do the same for the low homeowner's section. For the low auto deductible probability we get $\boxed{.18}$. For the low homeowner's deductible probability we get $\boxed{.19}$.

**c. What is the probability that the individual is in the same category for both auto and homeowner's deductibles?** For this we do more addition. We simply add up all the cells that have matching categories (ie: low and low, medium and medium). From that we get a $\boxed{.41}$ probability that an individual has the same category for both auto and homeowner's deductible.

**d. Based on your answer to part (c), what is the probability that the two categories are different?**
This one is straightforward. It's just all of the cells that aren't what we used in part c. Since everything in the table adds up to 1 we find that $1 - .42 = \boxed{.58}$.

**e. What is the probability that the individual has at least one low deductible level?**
To answer this we simply add up the low homeowner row and the low auto column while being careful to avoid duplicates. Doing so gives us $\boxed{.31}$.

**f. Using the answer in part (e), what is the probability that neither deductible level is low?**
Just like part (d) we simply do 1 minus our answer to (e). This gives us $\boxed{.69}$.

**Question 22.**

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The route used by a certain motorist in commuting to work contains two intersections with traffic signals. The probability that he must stop at the first signal is .4, the analogous probability for the second signal is .5, and the probability that he must stop at at least one of the two signals is .7. What is the probability that he must stop:

**a. At both signals?**
If we let stopping at the first signal be $A$ and the second signal be $B$, then we can calculate this using the general addition rule using the same strategy as in problem 14.

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\ P(A \cap B) &= .4 + .5 - .7 \\ P(A \cap B) &= .2 \end{aligned}$$

**b. At the first signal but not at the second one?**
This one is just the union of the two events minus the probability of $B$.

$$\begin{aligned} P(A \cap B') &= P(A \cup B) - P(B) \\ P(A \cap B') &= .7 - .5 \\ P(A \cap B') &= .2 \end{aligned}$$

**c. At exactly one signal?**
Here we simply add $P(A \cap B')$ and $P(B \cap A')$ together. We can assume the same strategy was used to calculate the latter probability. $.2 + .3 = \boxed{.5}$.

**Question 26.**

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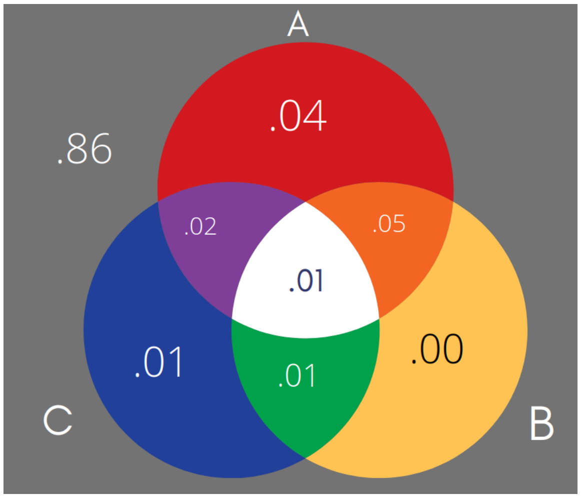
A certain system can experience three different types of defects. Let A, B and C denote the event that the system has a defect of each respective type. Suppose that

$$P(A) = .12 \qquad P(B) = .07 \qquad P(C) = .05$$
$$P(A \cup B) = .13 \qquad P(A \cup C) = .14 \qquad P(B \cup C) = .10$$
$$P(A \cap B \cap C) = .01$$

Before we even tackle the given problems we us calculate our intersections so we can create a Venn diagram. That will make this very smooth for us. We'll be using the general addition rule as usual.

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$
$$P(A \cap B) = .12 + .07 - .13$$
$$P(A \cap B) = .06$$

A similar strategy is used for the other two intersections which resulted in $P(A \cap C) = .03$ and $P(B \cap C) = .02$. From here we can create our diagram and we take note of our given triple intersection value when putting in our other intersections.




Now we can easily answer any questions provided.

**a. What is the probability that the system does not have a type A defect?**
We simply need to subtract out .12 from 1 to remove all type A defects from our probability. This gives us $\boxed{.88}$

**b. What is the probability that the system has both type A and type B defects?**
We can use our calculation for $P(A \cap B)$ here which was $\boxed{.06}$.

**c. What is the probability that the system has both type A and type B defects but not a type C defect?**
This can be defined as $P((A \cap B) \cap C')$. This is simply .06 from part (b) minus .01 from our triple intersection. The answer is $\boxed{.05}$.

**d. What is the probability that the system has at most two of these defects?**
This one is deceptively simple. Since the highest amount of defects we can have is 2, we just subtract out the probability that we get all 3 defects. Our answer here is $\boxed{0.99}$.

# Section 2.3: 30-31, 34-35, 39

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## Important notes and definitions for Section 2.3

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**Proposition:** If the first element or object of an ordered pair can be selected in $n_1$ ways, and for each of these $n_1$ ways the second element of the pair can be selected in $n_2$ ways, the number of pairs is $n_1 \cdot n_2$.

**Product Rule for k-Tuples:** Suppose a set consists of ordered collections of k elements (k-tuples) and that there are $n_1$ possible choices for the first element; for each choice of the first element, there are $n_2$ possible choices of the second element. For each possible choice of the first $k-1$ elements, there are $n_k$ choices of the $k$th element. Then there are $n_1 \cdot n_2 \cdot ... \cdot n_k$ k-tuples.

**Definition: Permutation / Combination:** A **permutation** is ordered subset. The number of permutations of size $k$ that can be formed from the $n$ individuals or objects in a group will be denoted by $P_{k,n}$. A **combination** is an unordered subset. One type of notation is $C_{k,n}$, but another more common notation is $\binom{n}{k}$, read "n choose k".

**Propositions:**

$$P_{k,n} = \frac{n!}{(n-k)!}$$

$$\binom{n}{k} = \frac{P_{k,n}}{k!} = \frac{n!}{k!(n-k)!}$$

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**Question 30.**

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A friend of mine is giving a dinner party. His current wine supply includes 8 bottles of zinfandel, 10 of merlot and 12 of cabernet (he only drinks red wine), all from different wineries.

**a. If he wants to serve 3 bottles of zinfandel and serving order is important, how many ways are there to do this?**

The answer would be the result of $P_{3,8}$, or 8 nPr 3, which is 336 permutations.

**b. If 6 bottles of wine are to be randomly selected from the 30 for serving, how many ways are there to do this?**

For this we can assume order isn't important, so we'll calculate 30 nCr 6, which is 593,775.

**c. If 6 bottles are randomly selected, how many ways are there to obtain two bottles of each variety?**

We'll make use of the multiplication rule here. We'll first see how many combinations of two of each specific type of alcohol we have and then multiply those together.

$$\binom{8}{2} \cdot \binom{10}{2} \cdot \binom{12}{2} = 83160$$

**d. If 6 bottles are randomly selected, what is the probability that this results in two bottles of each variety being chosen?**

Here we simply take our result from part (c) and divide it by our result from part (b).

$$\frac{83160}{593775} \approx .140$$

**e. If 6 bottles are randomly selected, what is the probability that all of them are the same variety?**

So all these events are mutually exclusive so we can add them together. We'll use P(A), B and C and let P(A) = probability of 6 bottles of zinfandel. So each letter will correspond to its respective type of alcohol.

$$P(A) + P(B) + P(C)$$
$$\frac{\binom{8}{6}}{\binom{30}{6}} + \frac{\binom{10}{6}}{\binom{30}{6}} + \frac{\binom{12}{6}}{\binom{30}{6}} \approx 0.00196$$

**Question 31.**

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The composer Beethoven wrote 9 symphonies, 5 piano concertos, and 32 piano sonatas.

**a. How many ways are there to play first a Beethoven symphony and then a Beethoven piano concerto?**
We can just simply multiply the number of symphonies by the number of concertos in this case. So, $\boxed{9 \cdot 5 = 45}$ ways of organizing those compositions.

**b. The manager of a radio station decides that on each successive evening (7 days per week), a Beethoven symphony will be played followed by a Beethoven piano concerto followed by a Beethoven piano sonata. For how many years could this policy be continued before exactly the same program would have to be repeated?**
Here we can do the same thing as part (b), but also multiply by the number of piano sonatas (32). Then, since we will have the number of days we can divide by 365 to roughly approximate the years this would last for.

$$9 \cdot 5 \cdot 32 = \boxed{1440 \text{ days } \approx 4 \text{ years}}$$

**Question 34.**

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Computer keyboard failures can be attributed to electrical defects or mechanical defects. A repair facility currently has 25 failed keyboards, 6 of which have electrical defects and 19 of which have mechanical defects.

**a. How many ways are there to randomly select 5 of these keyboards for a thorough inspection (without regard to order)?**
This one is simple. It's 25 nCr 5 which is $\boxed{53130}$.

**b. In how many ways can a sample of 5 keyboards be selected so that exactly two have an electrical defect?**
Here we multiply the combinations of our electrical keyboards in pairs by the other 19 keyboards in combinations of 3.

$$\binom{6}{2} \cdot \binom{19}{3} = \boxed{14525}$$

**c. If a sample of 5 keyboards is randomly selected, what is the probability that at least 4 of these will have a mechanical defect?**

$$\frac{n(E)}{n(S)} = \frac{\binom{6}{4} \cdot \binom{21}{1}}{\binom{25}{5}} \approx \boxed{0.00593}$$

**Question 35.**

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A production facility employs 10 workers on the day shift, 8 workers on the swing shift, and 6 workers on the graveyard shift. A quality control consultant is to select 5 of these workers for in-depth interviews. Suppose the selection is made in such a way that any particular group of 5 workers has the same chance of being selected as does any other group (drawing 5 flips without replacement from among 24).

**a. How many selections result in all 5 workers coming from the day shift? What is the probability that all 5 selected workers will be from the day shift?**

$$\frac{\binom{10}{5}}{\binom{24}{5}} \approx \boxed{0.0059}$$

**b. What is the probability that all 5 selected workers will be from the same shift?**
Let A = all 5 day, B = all 5 swing and C = all 5 grave. These are mutually exclusive so we add them together.

$$P(A) + P(B) + P(C) = \text{Answer}$$
$$\frac{\binom{10}{5}}{\binom{24}{5}} + \frac{\binom{8}{5}}{\binom{24}{5}} + \frac{\binom{6}{5}}{\binom{24}{5}} \approx \boxed{0.0074}$$

**Question 39.**

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A box in a supply room contains 15 compact fluorescent lightbulbs, of which 5 are rated 13-watt, 6 are rated 18-watt, and 4 are rated 23-watt. Suppose that three of these bulbs are randomly selected.

**a. What is the probability that exactly two of the selected bulbs are rated 23-watt?**

$$P(E) = \frac{n(E)}{n(S)} = \frac{\binom{4}{2} \cdot \binom{11}{1}}{\binom{15}{3}} \approx \boxed{.145}$$

**b. What is the probability that all three of the bulbs are the same rank?**
Our three possibilities are (All A), (All B) and (All C). So we want the sum of the probabilities of these three possibilities.

$$P(AllA) + (AllB) + (AllC)$$
$$\frac{\binom{5}{3}}{\binom{15}{3}} + \frac{\binom{6}{3}}{\binom{15}{3}} + \frac{\binom{4}{3}}{\binom{15}{3}} \approx \boxed{0.0747}$$

## Section 2.4, Conditional Probability: 47-50, 53-55, 59-60

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**Question 47.**

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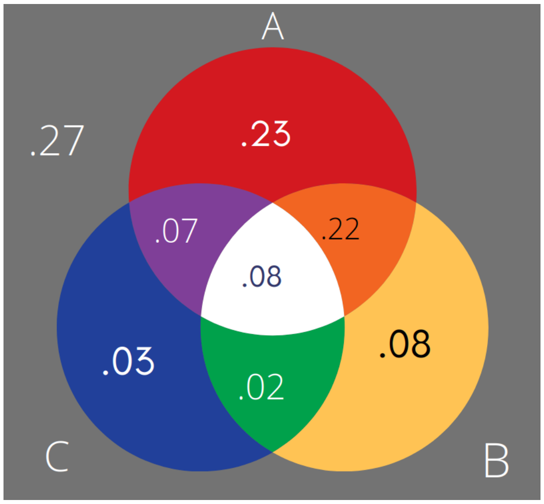
Return to the credit card scenario of Exercise 12 (Section 2.2), and let $C$ be the event that the selected student has an American Express card. In addition to $P(A) = .6$, $P(B) = .4$, and $P(A \cap B) = .3$, suppose that $P(C) = .2$, $P(A \cap C) = .15$, $P(B \cap C)$, and $P(A \cap B \cap C) = .08$.

Before we get started, let's make a Venn diagram that we can utilize. I would explain where the smaller intersections came from better, but I'm running out of time to transcribe these notes! Simply put, I took the given intersections and removed the triple intersection to get the values.




**a. What is the probability that the selected student has at least one of the three types of cards?**
$P(A \cup B \cup C)$ is the inside of the venn diagram all added together, what we get from this is $\boxed{= 0.74}$

**b. What is the probability that the selected student has both a Visa card and a Mastercard but not an American Express card?**
$P(A \cap B \cap C')$ is what we're solving for here. Using the venn diagram, we know this is part of our intersections and it's our one intersection that doesn't involve C. So, we get $\boxed{.22}$.

**c. Calculate and interpret $P(B|A)$ and also $P(A|B)$.**

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{0.3}{0.6} = \boxed{.5}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.3}{0.4} = \boxed{.75}$$

What we can say from these two calculations is that if a student has a Visa card, there is a 50% chance that they also have a MasterCard. We can also say that if a student has a MasterCard, there is a 75% chance that they have a Visa.

**d. If we learn that the selected student has an American Express card, what is the probability that she or he also has both a Visa card and a MasterCard?**

$$P(A \cap B|C) = \frac{P(A \cap B \cap C)}{P(C)} = \frac{.08}{.2} = \boxed{.4}$$

**e. Given that the selected student has an American Express card, what is the probability that she or he has at least one of the other two types of cards?**

$$P(A \cup B|C) = \frac{P((A \cup B) \cap C)}{P(C)} = \frac{.07 + .08 + .02}{.2} = \boxed{.85}$$

**Question 49.**

---

The accompanying table gives information on the type of coffee selected by someone purchasing a single cup at a particular airport kiosk.

| | **Small** | **Medium** | **Large** |
|---|---|---|---|
| **Regular** | 14% | 20% | 26% |
| **Decaf** | 20% | 10% | 10% |

Consider randomly selecting such a coffee purchaser.

**a. What is the probability that the individual purchased a small cup? A cup of decaf coffee?**
$a_i$**:** $.14 + .20 = \boxed{.34}$. $a_{ii}$**:** $.20 + .10 + .10 = \boxed{.40}$.

**b. If we learn that the selected individual purchased a small cup, what now is the probability that they chose decaf coffee, and how would you interpret this probability?**
For ease of writing, let us refer to cup sizes as $A_i$ with $A_1$ referring to small and so on. Let us refer to regular coffee as $B_1$ and decaf as $B_2$ in the same way.

$$P(B_2|A_1) = \frac{P(B_2 \cap A_1)}{P(A_1)} = \frac{.20}{.34} \approx \boxed{.588}$$

**c. If we learn that the selected individual purchased decaf, what now is the probability that a small size was selected, and how does it compare to the corresponding unconditional probability of (a)?**

$$P(A_1|B_2) = \frac{P(B_2 \cap A_1)}{P(B_2)} = \frac{.20}{.4} \approx \boxed{.5}$$

What we see here is that the probability is lower than that in part(b).

**Question 50.**

---

A department store sells sport shirts in three sizes (small, medium and large), three patterns (plaid, print, and stripe), and two sleeve lengths (long and short). The accompanying tables give the proportions of shirts sold in the varying category combinations.

| **Short-sleeved** | | | |
|---|---|---|---|
| | **Pattern** | | |
| *Size* | **Pl** | **Pr** | **St** |
| **S** | .04 | .02 | .05 |
| **M** | .08 | .07 | .12 |
| **L** | .03 | .07 | .08 |

| **Long-sleeved** | | | |
|---|---|---|---|
| | **Pattern** | | |
| *Size* | **Pl** | **Pr** | **St** |
| **S** | .03 | .02 | .03 |
| **M** | .10 | .05 | .07 |
| **L** | .04 | .02 | .08 |

**a. What is the probability that the next shirt sold is a medium, long-sleeved, print shirt?**
For these problems I will simply be pulling my answers from the tables provided. $\boxed{.05}$

**b. What is the probability that the next shirt sold is a medium print shirt?**

$$.07 + .05 = \boxed{.12}$$

**c. What is the probability that the next shirt sold is a short-sleeved shirt? A long-sleeved shirt?**
For this part I simply added up all the values in each respective table.
Short sleeve? $\boxed{.56}$
Long sleeve? $\boxed{.44}$

**d. What is the probability that the size of the next shirt sold is medium? That the pattern of the next shirt sold is a print?**
I added up all medium values for this one. It gave me $\boxed{.49}$.
As for the prints, $.16 + .09 = \boxed{.25}$.

**e. Given that the shirt just sold was a short-sleeved plaid, what is the probability that its size was medium?**

$$P(\text{Med} \mid \text{SS} \cap \text{Pl}) = \frac{\text{Med } \cap \text{ SS} \cap \text{Pl}}{P(\text{SS} \cap \text{Pl})} = \frac{.08}{.15} \approx \boxed{.533}$$

**f. Given that the shirt just sold was a medium plaid, what is the probability that it was short-sleeved? Long-sleeved?**

$$P(\text{SS} \mid \text{Med} \cap \text{Pl}) = \frac{\text{Med } \cap \text{ SS} \cap \text{Pl}}{P(\text{Med} \cap \text{Pl})} = \frac{.08}{.12} \approx \boxed{.667}$$

$$P(\text{LS} \mid \text{Med} \cap \text{Pl}) = \frac{\text{Med } \cap \text{ LS} \cap \text{Pl}}{P(\text{Med} \cap \text{Pl})} = \frac{.08}{.17} \approx \boxed{.471}$$

**Question 53.**

---

A certain shop repairs both audio and video components. Let A denote the event that the next component brought in for repair is an audio component, and let B be the event that the next component is a compact disc player (so the event B is contained in A). Suppose that $P(A) = .6$ and $P(B) = .05$. What is $P(B|A)$?

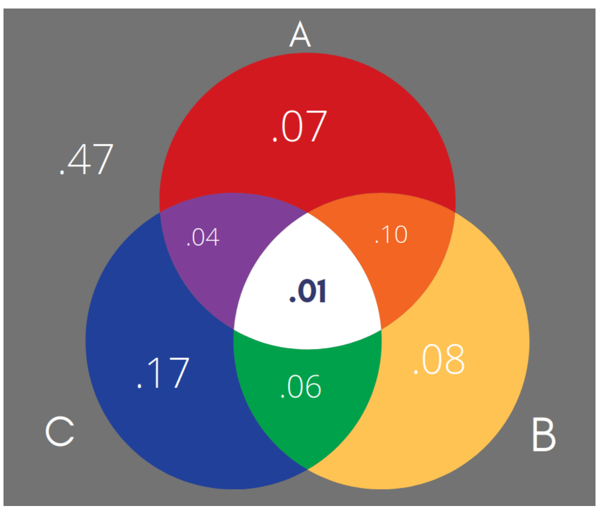
$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{.05}{.6} \approx \boxed{.0833}$$

**Question 54.**

---

In Exercise 13, $A_i = \{\text{awarded project i}\}$ for $i = 1, 2, 3$. Use the probabilities given there to compute the following probabilities, and explain in words the meaning of each one.

$$\begin{aligned} P(A) &= .22 \\ P(B) &= .25 \\ P(C) &= .28 \\ P(A \cap B) &= .11 \\ P(A \cap C) &= .05 \\ P(B \cap C) &= .07 \\ P(A \cap B \cap C) &= .01 \end{aligned}$$




**a.** $P(B|A)$

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{.11}{.22} = \boxed{.5}$$

**b.** $P(B \cap C|A)$

$$P(B \cap C|A) = \frac{P(B \cap C \cap A)}{P(A)} = \frac{.01}{.22} \approx \boxed{.045}$$

**c.** $P(B \cup C|A)$

$$\begin{aligned} P(B \cup C) &= .25 + .28 - .07 = .46 \\ P((B \cup C) \cap A) &= .15 \\ P(B \cup C|A) &= \frac{P((B \cup C) \cap A)}{P(A)} \\ &= \frac{.15}{.22} \approx \boxed{.682} \end{aligned}$$

**d.** $P(A \cap B \cap C \mid A \cup B \cup C)$

$$P(A \cap B \cap C \mid A \cup B \cup C) = \frac{P((A \cap B \cap C) \cap (A \cup B \cup C))}{P(A \cup B \cup C)} = \frac{.01}{.53} \approx \boxed{.0189}$$

**Question 55.**

---

Deer ticks can be carriers of either Lyme disease or human granulocytic ehrlichiosis (HGE). Based on a recent study, suppose that 16% of all ticks in a certain location carry Lyme disease, 10% carry HGE, and 10% of the ticks that carry at least one of these diseases in fact carry both of them. If a randomly selected tick is found to have carried HGE, what is the probability that the selected tick is also a carrier of Lyme disease?

Let P(A) = the percent that carry lyme disease and P(B) be the ones with HGE. My entire solution will just be algebra here, so I'll let it speak for itself.

$$
\begin{aligned}
P((A \cap B) \mid A \cup B) &= .1 \\
P((A \cap B) \mid A \cup B) &= \frac{P((A \cap B) \cap (A \cup B))}{P(A \cup B)} = .1 \\
P((A \cap B) \cap (A \cup B)) &= P(A \cap B) \text{ (by inspection)} \\
\frac{P(A \cap B)}{P(A \cup B)} &= 0.1 \\
P(A \cap B) &= 0.1 \cdot P(A \cup B) \\
P(A \cap B | B) &= \frac{A \cap B}{P(B)} \\
&= \frac{0.1 \cdot P(A \cup B)}{0.1} \\
P(A \cap B | B) &= P(A \cup B) \\
P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\
P(A \cup B) &= P(A) + P(B) - \frac{P(A \cup B)}{10} \\
P(A \cup B) + \frac{P(A \cup B)}{10} &= P(A) + P(B) \\
P(A \cup B)\left(1 + \frac{1}{10}\right) &= .16 + .10 \\
P(A \cup B) &= \frac{.26}{1.1} \approx 0.236 \\
P(A \cap B \mid B) &= \boxed{0.236}
\end{aligned}
$$

**Question 59.**

---

At a certain gas station, 40% of the customers use regular gas ($A_1$), 35% use plus gas ($A_2$), and 25% use premium ($A_3$). Of those customers using regular gas, only 30% fill their tanks (event B). Of those customers using plus, 60% fill their tanks, whereas of those using premium, 50% fill their tanks.

| | | |
|---|---|---|
| $P(A_1) = .40$ | $P(A_2) = .35$ | $P(A_3) = .25$ |
| $P(B \mid A_1) = .3$ | $P(B \mid A_2) = .6$ | $P(B \mid A_3) = .5$ |
| $P(A_1 \cap B) = .12$ | $P(A_2 \cap B) = .21$ | $P(A_3 \cap B) = .125$ |

Note: Intersections calculated using Baye's theorem. I will spare you the calculations.

**a. What is the probability that the next customer will request gas and fill the tank** $(A_2 \cap B)$?

$$P(B \mid A_2) = \frac{P(B \cap A_2)}{P(A_2)}$$

$$P(B \cap A_2) = P(B \mid A_2) \cdot P(A_2)$$

$$P(B \cap A_2) = .6 \cdot .35 = \boxed{.21}$$

**b. What is the probability that the next customer fills the tank?**
This part will utilize the law of total probability. I will spare you the plugging in of every individual part of this.

$$P(B) = \sum_i P(B \mid A_i) \cdot P(A_i) = \boxed{.455}$$

**c. If the next customer fills the tank, what is the probability that regular gas is requested? Plus? Premium?**

$$P(A_1) = \frac{.12}{.455} \approx \boxed{.264}$$

$$P(A_2) = \frac{.21}{.455} \approx \boxed{.462}$$

$$P(A_3) = \frac{.125}{.455} \approx \boxed{.264}$$

**Question 70.**

---

Reconsider the credit card scenario of Exercise 47 (section 2.4), and show that $A$ and $B$ are dependent first by using the definition of independence and then by verifying that the multiplication property does not hold.

Based on the definition of independence, it's true if and only if $P(A \mid B) = P(A)$. Pulling our numbers from Exercise 47, we can see that $.75 \neq .6$. Therefore, we know these are dependent on each other. As for the multiplication rule, we know we have independence if and only if $P(A \cap B) = P(A) \cdot P(B)$. Since $.3 \neq .6 \cdot .4$, we know we do not have independence.